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The development and importance of proficiency in basic calculation

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1. Summary

- Proficiency in basic calculation (single digit addition and subtraction) is the strongest correlate of success in mathematics in primary school.
- It develops in conjunction with understanding of the number system and number principles.
- In a sample of schools judged average to outstanding by Ofsted inspectors and with above average performance in Key Stage 2 tests, teachers agreed that developing basic calculation proficiency was important.
- Nevertheless, the current English National Curriculum goal for knowledge of addition and subtraction facts was achieved by none of the 259 children when they were assessed either in Year 3 or Year 4.
- Despite ignorance of many facts, their mathematical achievement was slightly above average.
- Incomplete knowledge of number facts is not the barrier to success in mathematics it is often believed to be.

2. Implications

- Conceptual knowledge plays more of a part in developing fluency in simple calculation than advocates of learning facts by memorizing tables would expect.
- The current National Curriculum goal of complete knowledge of basic calculation facts by Year 3 is unlikely to be achieved without substantial change in the way children are taught.
- What works best to develop children's fluency and whether it would yield benefits in general mathematical achievement are questions that require experimental investigation.

3. Background

Children differ hugely in their mathematical performance at the end of primary school. At any point in primary schooling the differences between individuals in the *same* school exceed those between schools or countries.

As performance at primary school predicts subsequent mathematical achievement and the ability to manage the numerical demands of everyday life in adulthood, understanding what causes this variation and what will help primary children develop could yield substantial long term benefits.

Previous research has identified many factors as playing a part in accounting for differences in children's attainment. They include family and school characteristics but child characteristics are the ones that show the strongest relationships.

Child level variables that are related to mathematical performance include cognitive factors that correlate with general educational achievement (general ability, memory functioning, processing speed, and oral language) and socio-emotional functioning. These affect both mathematics and reading. This partly explains why children who are doing well in maths are typically doing well in reading, and why difficulties in maths and reading frequently go together.

Basic calculation proficiency is skill in solving the addition of whole numbers with sums less than 20 and corresponding subtractions. Although it is a small part of arithmetic, and arithmetic is just one

aspect of mathematics, basic calculation proficiency shows substantial covariation with more general measures of mathematics performance. Deficiencies in basic calculation are the commonest characteristic of children making poor progress.

Previous research has also identified aspects of conceptual knowledge (understanding of number principles and the number system) as accounting for differences between primary children in mathematics achievement.

4. Educational views

In education there are different views of basic calculation proficiency and how to develop it. The *traditional* view equates proficiency with having the solutions to basic calculations stored in long term memory as number facts that can be quickly and accurately retrieved. Its proponents advocate learning tables of facts and practice. The *traditional* view was dominant in the UK during the nineteenth and early twentieth century.

The *progressive* view equates proficiency with rapid and accurate solution of problems. Its proponents consider learning numerical principles and patterns and knowing how to use them efficiently and accurately is more important for mathematical progress than memorizing isolated facts.

The basic calculation problem *7 – 6* illustrates the difference between the *traditional* and *progressive* views of proficiency. In the *traditional* view being able to answer this problem quickly results from having learnt *7 – 6 = 1* as a fact. In the *progressive* view a quick answer results from understanding subtraction and the numerical relation between adjacent counting numbers. Proponents of the *progressive* view claim that these understandings would also enable quick answers to *153 – 152* and *2365 - 2364*.

The current English National Curriculum takes a *consensus* view. Like the *traditional* view, this regards knowledge of facts as essential for proficiency. However, it assumes that knowledge of facts will be developed through using number principles to expand knowledge of facts. By using their knowledge that *6 + 1 = 7* and their understanding of the relation between addition and subtraction, a child can derive the answer to *7 - 6*. With practice, the answer to *7 - 6* becomes so well known that the child needs no longer to work it out.

5. Why mathematical performance is related to basic calculation proficiency

The educational views explain the relation between basic calculation proficiency and more general mathematical performance differently. Both the *traditional* and *consensus* views maintain that basic calculation proficiency is necessary for mathematical progress: unless a child knows the facts they will be at a disadvantage. The *progressive* view maintains that knowledge of principles and patterns is important for both.

Psychologists suggest that quite apart from the logical connections between simple and complex arithmetic, the same factors are likely to affect the development of basic calculation proficiency and more general mathematical progress.

6. Aims

The aims of our ESRC-funded study were to examine (a) how basic calculation proficiency develops from Year 3 to Year 4, (b) how basic calculation proficiency varies with conceptual knowledge, and (c) what factors explain the relation between basic calculation proficiency and general mathematics achievement (Cowan, Donlan, Shepherd, Cole-Fletcher, Saxton, & Hurry, in press).

7. Methods

7.1 Participants

We recruited seven maintained schools in the Royal Borough of Windsor and Maidenhead. Overall our sample of schools was above average in attainment on national tests and had received positive Ofsted reports. The 259 children who participated were less disadvantaged than average, as indicated by take up of free school meals, residential postcodes, and socio-economic classifications of households based on interviews of parents.

7.2 Measures

Each pupil was individually assessed using tests of basic calculation proficiency and conceptual knowledge (understanding of calculation principles and the number system) in both Year 3 and 4. Standardized tests were also used to assess general ability, memory functioning, processing speed, oral language, reading and mathematics.

Teachers rated their pupils' socio-emotional functioning on a widely used questionnaire. They also talked about their beliefs in the importance of basic calculation proficiency and how children acquire it.

8. Main findings

8.1 How basic calculation proficiency developed

- Basic calculation proficiency in the sample was much below National Curriculum expectations that children should know all the answers to basic calculation problems by the end of Year 3.
- Rapid correct answers to basic calculation problems increased from 45% to 58%.
- No child used number facts to solve every problem and only two pupils (< 1%) answered all basic calculation problems rapidly and accurately.
- Even in Year 4 only about a third of the children (34%) solved all the basic problems correctly using *any* method.

8.2 The connections between basic calculation proficiency and conceptual knowledge

- Basic calculation proficiency and conceptual knowledge support each other.
- Year 3 levels of both predicted Year 4 success in both, even when other factors were controlled.

8.3 Explaining the relation between basic calculation proficiency and general mathematics achievement

- Contrary to both *traditional* and *consensus* views, the sample's maths achievement was above average despite much ignorance of facts.
- Contrary to the *progressive* view, the relation between basic calculation proficiency and maths achievement was only partly explained by associations with conceptual knowledge and general cognitive factors.
- Basic calculation proficiency explained most variation in mathematics achievement but conceptual knowledge and cognitive factors were also important.

9. Next steps

There are many teaching interventions (e.g. Askew, Bibby, & Brown, 2001), private tutorial systems (e.g. Kumon) and computer games (e.g. Dr Kawashima's Brain Training) that claim to enhance children's basic calculation proficiency but they have not been rigorously assessed (Dowker, 2004) and the benefits for children's general mathematical achievement remain uncertain. Our study indicates that such research is needed.

10. References

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